

Hedge Funds, Active Management, and the Asset Allocation Decision:
A Descriptive Framework

September 2005

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Introduction

Over the last several years, hedge funds have grown dramatically, both in terms of the number of funds and the total assets under management. In some cases, the funds have been able to deliver returns well in excess of those that were available from more traditional investments; in other cases, they have experienced extremely large losses. Given this wide range of experiences, it is only natural that a great debate has ensued as to what is the appropriate role of hedge funds in an investor's portfolio. One way to frame this debate is from an asset allocation perspective: should hedge funds be viewed as a separate and distinct asset class or as simply a form of active management?

To establish the appropriate characterization, it is necessary to consider the sources of return and the sources of risk that comprise an investment in hedge funds and how these sources of return and risk fit into the overall asset allocation decision.¹ In particular, do the return and risk characteristics of the hedge fund universe support the treatment as an asset class and, if so, how should that asset class be characterized? From an asset allocation perspective, an asset class should represent a potential source of return that offers investors a sufficient long-term return premium from a (relatively) passive investment. The portfolio representing the asset class should be investable and its composition should change minimally, if at all, over time. Other than identifying the common source of systematic risk that is responsible for the expected return premium, the construction of the portfolio should not require any of the skills and judgments typically associated with active management.

¹ Numerous studies have tried to identify persistent systematic sources of return and risk among hedge funds. See, for example, Schneeweis and Spurgin (1998), Mitchell and Pulvino (2000), Fung and Hsieh (2001), Brealey and Kaplanis (2001), Brown and Goetzmann (2001) Schneeweis, Kazemi, and Martin (2002), Dor and Jagannathan (2002), and Agarwal and Naik (2003). As discussed below, hedge funds will almost never be the low-cost provider of systematic returns and their inclusion in an investor's portfolio should be dependent on expected returns that can not be replicated with long-term, passive investments.

If hedge funds are to be thought of as an asset class, then it must follow (at least in theory) that a passive benchmark or index can be constructed that captures the significant sources of return and risk that would define the asset class. If, on the other hand, hedge funds are just a form of active management, then it will not be optimal to construct a passive benchmark or index because an optimally constructed portfolio of active managers would not be all inclusive and would require the application of skill-based judgments in the portfolio construction.

One of the biggest challenges in evaluating the appropriate role of hedge funds in the asset allocation decision may simply be to define what they are and, more importantly, what they are supposed to be. Given the breadth of hedge fund strategies and the limited regulation of the funds, identifying a stable and unique set of return and risk characteristics that accurately describes a potential investment in hedge funds is likely to be impossible, especially if the goal is to identify sources of risk that can be expected to earn a significant return premium from a long-term asset allocation perspective.

In fact, one can argue persuasively that the only two important characteristics that consistently apply to hedge funds are, (i) they are subject to minimal investment constraints and, (ii) they charge high fees. It is certainly true that the typical hedge fund is much less constrained in terms of its potential portfolio holdings than traditional asset managers. They tend to have the ability to invest in a wide range of securities, to purchase and sell derivative instruments based on other securities, to hold negative positions in securities through short sales, and to borrow or use other forms of leverage.²

² In his thought-provoking look at the future of investment management, Bernstein (2003) highlights the importance of investor flexibility for active managers and the advantage that this potential flexibility offers to hedge fund managers. See Ennis (2004) for an excellent discussion of the potential implications of such flexibility for active management, from an asset allocation perspective.

It is also true that the typical hedge fund tends to charge fees that are much higher than those charged by traditional asset managers. These fees usually include a performance-based component that pays the manager a percentage of the profit (or the profit in excess of the return that could be earned by investing in the riskless asset or some other benchmark), without any requirement that the manager also participate in any losses.

Given these characteristics, this analysis will attempt to show that hedge funds should be viewed as simply a potential source of active management, attempting to exploit skill-based forecasting ability. From this perspective, the appropriate role for hedge funds in an investor's overall asset allocation can be determined according to the desired amount of active management in the portfolio. This is separate from the passive allocation which constitutes the policy portfolio, and which, by definition, will comprise systematic sources of return and risk, or "beta." The desired amount of active management will depend on the potential differences in the expected returns and risks in the overall (passive plus active) portfolio resulting from the pursuit of "alpha," taking into account the increased flexibility available to hedge fund managers.³

For all active managers, skill-based performance can and should be separated from the returns available from passive investments.⁴ This separation results in an active portfolio that includes all of the investments that are the result of market-timing or security-selection decisions and nothing else. This active portfolio reflects the complete separation of the skill-based investment decisions from the appropriate benchmark or passive

³ See Waring and Siegel (2003) for an excellent discussion of the role of active management in an investor's overall portfolio.

⁴ Much of the original work in this area followed from the development of the Capital Asset Pricing Model. In this context, Fama (1972) presented a systematic disaggregation of investment returns, relative to a benchmark portfolio. Jensen (1968, 1969) used a CAPM framework to evaluate the performance of mutual funds return data. Sharpe (1992) extended this analysis, using a multi-factor model to separate the returns of mutual funds that are the result of skill-based active management from those returns that could have been replicated with passive investments in index funds.

portfolio. This separation creates two advantages: first, it allows *all* investment managers to be evaluated relative to each other, and; second, it allows hedge fund managers, and all other active managers for that matter, to be evaluated as a potential active management overlay on the policy portfolio within the context (or framework) of the asset allocation decision.

Because investment decisions related to skill-based active management can be separated from any passive policy portfolio, all active management investments must be treated as if they require self-financing. The desired amount of active management can be implemented separate from the allocation of investment capital and separate from any applicable policy portfolio. This result is tautological since the asset allocation process must allocate 100 percent of all investment capital to the passive investments that comprise the policy portfolio.

It is therefore also tautological that the alpha generated from active management must be portable; that is, it can be combined with any of the asset classes (including cash) that comprise the policy portfolio as part of the asset allocation decision. The investment universe that a manager uses to generate the alpha from skill-based investments does not need to be restricted to the asset class investments defined by the asset allocation process that led to the policy portfolio.

As the desired amount of active management can be implemented separate from the allocation of investment capital, the concept of leverage is arbitrary. The desired leverage for an investment portfolio can only be determined after the optimal level of risk for the overall portfolio, given both the passive and active investment opportunities, is established. Since all active management investments must be evaluated as if they require self-financing, the assignment of capital to finance them from allocated assets in the policy portfolio will be arbitrary. What will matter is the incremental return that can be

expected from active management and the amount of corresponding incremental risk the active management generates. The desired amount of active management can be implemented as an overlay to the policy portfolio.

Therefore, the appropriate exposure to the returns and risks from active management can be determined separate from the determination of the policy portfolio, but within the context of the asset allocation decision. Constraints, however, will be a function of the risk budget, rather than the allocation of capital.⁵

This does not mean that the leverage used in an active management investment does not matter. What it does mean is that the impact of leverage on the expected returns and risks of a portfolio can be estimated and then incorporated into the portfolio construction process and, thereby, into the decision with respect to the desired amount of active management in the overall portfolio. What matters from an asset allocation perspective is not how much capital to allocate to active management, but how much incremental expected return and risk associated with the active management should be added to the policy portfolio.

Since all active management investments must be treated as if they require self-financing, all returns from active management can be defined as the returns from taking risk. From this perspective, all of the components of an active manager's return can be expressed as returns net of the riskless rate, or net of the financing rate. All of the components of the return will be incremental to the return that could be earned without risk: the return from investing in cash. Returns from active management will simply

⁵ This said, gaining the appropriate exposure to the returns and risks of active management is not a trivial task. The composition of an optimally constructed portfolio of active managers requires skill-based abilities that are similar to those the active managers employ to generate alpha. Absent these skills, it will be impossible to determine the appropriateness of including an active manager in the optimal portfolio of active managers.

reflect the returns associated with the incremental risk exposure derived from holdings that deviate from the passive portfolio.

From an asset allocation perspective, index funds, explicitly managed to minimize costs, will dominate hedge funds as a provider of passive asset class returns. All investment managers can be thought of as investing in a combination of passive and active investments. Given their fee structure, hedge funds provide an extremely inefficient source of passive investment returns. Index funds charge significantly lower fees, have much lower costs, and provide greater liquidity and complete transparency. To avoid this inefficiency, hedge funds should be evaluated relative to the returns available from cash and not relative to any asset class benchmark. Hedge funds should not be considered to be a long-term provider of systematic returns or beta. Long-term, they should have, on average, no exposure to any of the passive asset classes, other than cash.

Given the flexibility available to hedge fund managers, their returns and risks can easily be structured to provide a pure play on skill-based active investment. All hedge fund investments, other than cash, should be viewed as active investment decisions. The increased flexibility available to hedge fund managers should allow those managers who possess active management skills to dominate managers with the same skill set, but less investment flexibility (at least before taking into account fee differences). In fact, the combination of high fees and increased investment flexibility should attract the best active managers to hedge funds.

Of course, the increased flexibility afforded the hedge fund manager also requires more frequent and sophisticated monitoring by the investor, both in terms of identifying active management skill and reducing the possibility that the increased flexibility will be used by the manager to take risk that is different from or in excess of what is expected by the

investor. Therefore, the manager selection process will be extremely important, especially given all of the empirical evidence that highlights the difficulty in achieving significantly positive and sustainable returns from alpha.⁶ In the same spirit, frequent performance evaluation and vigilant risk monitoring is required with any investment in a hedge fund.

Performance evaluation and risk monitoring of hedge fund investments requires the disaggregation of the returns generated by the hedge fund manager to better understand the sources of return and sources of risk in the active portfolio. By disaggregating the portfolio holdings of the active manager and targeting the applicable themes, strategies, or risks inherent in the portfolio, investors can obtain important insights into the sources of return and the accompanying sources of risk that are generated by the manager's investment decisions. While the desired level of disaggregation will certainly require much more information than is available from the monthly return series reported by the typical hedge fund, in most cases, managers or prime brokers should be able to generate a daily time series of disaggregated manager returns, using no more information than the minimum required by the manager for their own basic risk management. Even if only available with some time delay, such disaggregated returns should be able to provide substantial information for purposes of performance evaluation and risk monitoring, without requiring knowledge of exact security positions.

Given that the appropriate role of hedge funds should be as a pure play on skill-based active management, it follows that they should not be thought of as a potential asset

⁶ Since the path breaking papers of Jensen (1968, 1969) evaluating the performance of mutual funds, there have been numerous studies that have demonstrated the difficulty of active managers achieving positive, sustainable returns from active management. See for example, Henriksson (1984), Sharpe (1992), and Carhart (1997) for further analysis of mutual fund performance. While hedge fund returns have shown more promise, a number of recent studies have highlighted the difficulties involved in the statistical analysis of hedge fund returns. See for example, Brown, Goetzmann, and Ibbotson (1999), Fung and Hsieh (2000), and Posthuma and van der Sluis (2003).

class. An optimally constructed portfolio of hedge funds should be considered as part of the active management overlay on the policy portfolio within the context of the asset allocation decision. By definition, the return to active management in the aggregate must be zero, or negative when the costs and fees of the active management process are included.⁷ Therefore, there can not be any optimal aggregation of hedge funds that will provide a feasible, passive investment alternative. Just as with the selection of any active manager, the construction of an optimal portfolio of hedge funds will require an evaluation of each potential manager's skill-based forecasting ability as part of the manager selection process. And given the variability of skills possessed by hedge fund managers, it will never make sense to invest with all possible hedge funds. Even the construction of an index fund of hedge funds that offers an attractive return / risk profile from an asset allocation perspective will require the equivalent of active management skills in the index construction process. That is, it will depend on the ability of the index provider to identify hedge fund managers that offer a sustainable forward-looking source of positive alpha.

The decision as to whether, and if so, how much, to invest in hedge funds is no different than the decision with respect to how much active management is desirable in the portfolio. It will depend on the expected return from skill-based active management investments and the characteristics of the risks that the investor is required to bear to achieve those expected returns. It will *not* depend on the returns available from an investment spanning the universe of hedge funds, but will depend on the returns available from an optimally constructed portfolio of active managers. It will depend on the quality of manager selection process, the portfolio construction, and the ongoing risk monitoring of the active managers.

⁷ Sharpe (1991) provides a simple and elegant proof of this requirement for the aggregate returns of all active managers.

A Simple, Descriptive Framework for Investment Manager Evaluation

In order to evaluate the appropriate role for hedge funds in an investment portfolio, it is necessary to compare the characteristics applicable to hedge fund managers with those of other investment managers. This can be accomplished through the use of a simple, descriptive framework that can be applied to all investment managers.⁸ Such a framework requires minimal assumptions and can be used to highlight the implications of the two primary characteristics of hedge funds described above: they are subject to minimal investment constraints and they charge high fees. Many of the results from this analysis may appear self-evident. However, given the disparate manner in which hedge fund investments are treated by institutional investors in practice, it is a useful exercise to consider exactly how hedge funds compare to other forms of investment management.⁹

The framework is based on the portfolio holdings of an investment manager. While again perhaps stating the obvious, it is important to remember that the returns generated by any investment manager will come from a portfolio of investments that may be expected to change over time. The time period for such changes may range from seconds to years; but at any point in time, the then current composition of the portfolio will be known, at least to the manager.

For any investment manager, the investment portfolio of manager i , at time t , can be described as

⁸ While the terminology used may be most familiar for equity managers, the framework and the use of disaggregation is certainly applicable for all investment managers.

⁹ Sharpe expressed much the same sentiment in Sharpe (1991), when comparing the returns in the aggregate from active investors and passive investors. Responding to such quotes as “Today’s fad is index funds that track the Standard and Poor’s 500. True, the average soundly beat most stock funds over the past decade. But is this an eternal truth or a transitory one?” from the September 3, 1990 issue of *Forbes*, he wrote “To be sure, we have seriously belabored the obvious, but the ubiquity of statements such as those quoted earlier suggests that such labor is not in vain.”

$$(1) \text{ Portfolio Holdings (i, t)} = \sum_k w_{kit} (\text{Security } k) + w_{it} (\text{Cash})$$

where w_{kit} is the proportion of the investment capital invested in Security k , $w_{it} (\text{Cash})$ is the proportion of net cash held in the portfolio, and $\sum_k w_{kit} + w_{it} (\text{Cash}) = 1$.¹⁰

As described in (1), no restrictions are placed on w_{kit} , and the summation is assumed to span all possible investment securities, other than cash. As such, (1) is applicable to all investment managers. Everything that follows will be based on the disaggregation of this portfolio. By carefully disaggregating the portfolio holdings over time, it is possible to better evaluate all of the important return and risk characteristics of any manager's investment decisions. In the case of active managers, at minimum, this disaggregation will allow for the separation of returns attributable to exposure to systematic risk (beta) and returns attributable to skill-based investment decisions, resulting from either market-timing or security-selection decisions (alpha).

The disaggregation process should focus on potentially important sources of risk and expected return. As the performance of many managers is judged relative to a benchmark portfolio,¹¹ it is useful to subdivide a manager's portfolio as follows

$$(2) \text{ Portfolio Holdings (i, t)} = \beta_{Mit} * \{\text{Benchmark Portfolio Holdings (M,t)}\} +$$

¹⁰ This assumes that the manager is actually allocated any capital. As will be shown below, active management must be self-financing, thus, in theory, requiring no investment capital. In the case of such a pure-play on active management, it must follow that $\sum_k w_{kit} + w_{it} (\text{Cash}) = 0$.

¹¹ It is the current practice that most managers are evaluated relative to the performance of a benchmark portfolio. This is why they are called relative return managers. At minimum, the performance evaluation of a manager should take into account the level of interest rates, comparing the return earned from taking risk with the riskless return available to investors. From this perspective, even "absolute return" managers should be evaluated relative to the riskless rate or the return available without taking risk.

$$\{(Security\ Selection\ Portfolio\ (i,t)\} + w_{it}(Cash)$$

where $\{Benchmark\ Portfolio\ Holdings\ (M,t)\}$ represents the exact security composition of the benchmark portfolio M, at time t, and represents a *passive, investable* portfolio that comprises the following holdings

$$(3) \{Benchmark\ Portfolio\ Holdings\ (M,t)\} = \sum_k w_{Mkt} (Security\ k)$$

where w_{Mkt} is the proportion of the benchmark portfolio invested in Security k and it is required that $\sum_k w_{Mkt} = 1$.

β_{Mit} captures the manager's exposure to the benchmark portfolio at time t. It is a measure of systematic risk and is calculated by the ratio of the covariance of the return of manager i with the benchmark portfolio return divided by the variance of the benchmark portfolio return. If the manager's holdings are either in cash or in securities exactly replicating the benchmark portfolio, then β_{Mit} will simply equal the proportion of the investment capital invested in the benchmark portfolio. Such a portfolio would *not* include any security selection activity.

As such, β_{Mit} will equal the portfolio-weighted average of all of the individual security betas, relative to the benchmark portfolio (M), for all of the individual securities held in manager i's portfolio, as shown

$$(4) \beta_{Mit} = \sum_k w_{kit} * \beta_{Mkt}$$

where β_{Mkt} is the ratio of the covariance of the return of security k with the benchmark portfolio return divided by the variance of the benchmark portfolio return.¹²

{Security Selection Portfolio (i,t)} captures all security holdings in the portfolio at time t that deviate from their holdings in [β_{Mit} * {Benchmark Portfolio Holdings (M,t)}] and represents all skill-based security selection undertaken by the manager. Any investment decision that is security-specific in nature is included in the Security Selection Portfolio. This portfolio can be described as

$$(5) \text{ \{Security Selection Portfolio (i,t)\} = } \sum_k (w_{kit} - \beta_{Mit} * w_{Mkt}) \text{ (Security k)}$$

By construction, the performance of the Security Selection Portfolio will have zero correlation with the performance of the benchmark portfolio, as β_{Mit} is assumed to capture all of the exposure of manager i's portfolio to the systematic risk of the passive benchmark portfolio. The Security Selection Portfolio will therefore also have zero covariance and a zero beta, with respect to the benchmark portfolio. $w_{it \text{ (Cash)}}$ represents the net amount of cash that is left over from (or borrowed in addition to) the investment capital after implementing the desired benchmark exposure and security selection.¹³

¹² In practice, β_{Mit} and β_{Mkt} must be estimated and therefore will not be known exactly. This may introduce tracking error into the process of separating the desired benchmark exposure from security selection. However, this source of error should be small for most portfolios that are not heavily concentrated in a few securities. The estimates of β_{Mit} and β_{Mkt} are the types of inputs needed by a manager to implement the most basic risk analytics. In fact, the estimate of β_{Mit} represents the manager's expected exposure to the benchmark portfolio and would be exact if the manager's holdings matched the weightings of the benchmark portfolio. Therefore, it is quite reasonable to consider β_{Mit} as the manager's exposure to the benchmark and any portfolio return deviations to be the result of security selection.

¹³ As described in (2), all of the desired systematic exposure is captured in one benchmark portfolio. It is a simple extension to include multiple benchmark portfolios in (2), although most managers will be evaluated relative to only one benchmark portfolio. Multiple benchmarks will be more relevant from a market-timing perspective, as discussed below.

The manager's exposure to the benchmark portfolio can be disaggregated into two parts: b_{Mit} , which is the *prespecified* asset allocation target value of β_{Mit} for manager i at time t , and θ_{Mit} , which is defined as $(\beta_{Mit} - b_{Mit})$ and reflects the opportunistic or market-timing component of β_{Mit} .¹⁴ In combination, b_{Mit} and θ_{Mit} represent the proportion of the benchmark return, in excess of the riskless return, $(R_{Mt} - R_{ft})$, that is expected to be captured by the manager's portfolio over time period t . From an asset allocation perspective, b_{Mit} represents the desired long-term systematic exposure to the asset class represented by the benchmark portfolio. At any point in time, θ_{Mit} captures all incremental exposure to the benchmark portfolio (overweighted or underweighted) in the manager's portfolio, relative to b_{Mit} , the manager's prespecified asset allocation targeted exposure. As defined, θ_{Mit} can be positive or negative and there are no constraints on its magnitude. From a long-term perspective, the average value for θ_{Mit} should be expected to equal zero in order to avoid a systematic source of bias which would be at odds with the asset allocation decision.¹⁵

The manager's Security Selection Portfolio can also be disaggregated to reflect different strategies or themes. In particular, this disaggregation can be used to focus on different sources of return and risk in the portfolio and should be used for purposes of return and risk attribution.

¹⁴ This description of a manager's potentially changing exposure to the benchmark portfolio follows from Henriksson and Merton (1981). In this framework, b_{Mit} reflects the passive exposure from an asset allocation perspective and θ_{Mit} reflects the benchmark exposure that is the result of active management from market timing. Changes in θ_{Mi} will capture the market-timing decisions of the active manager. As discussed in Henriksson and Merton (1981), this framework can easily be extended to accommodate a multi-factor structure, with the appropriate adjustment in the measurement of the betas. This allows for multiple or disaggregated sources of market-timing activity. As most managers will only be compared to one passive portfolio, the appropriate passive exposure will be $b_{Pit} = 0$ for all other benchmark portfolios. Therefore, it follows for all other benchmark portfolios that $\beta_{Pkt} = \theta_{Pit}$ as all systematic exposure can be considered to be market-timing activity.

¹⁵ This will also be true for multiple or disaggregated sources of market-timing activity. The long-term average exposure to all sources of systematic risk as a result of market-timing activity should equal zero in order to avoid systematic biases that would be at odds with the asset allocation decision.

This will be especially important for purposes of performance evaluation. By disaggregating the different sources of return and risk, it will be possible to gain much better insight into investment decision process used by an active manager attempting to generate skill-based returns.¹⁶

A simple example of this disaggregation would be to separate the Security Selection Portfolio into two parts: {Long Portfolio (i,t)}, which includes all security holdings that are overweighted relative to $[(b_{Mit} + \theta_{Mit}) * \{\text{Benchmark Portfolio Holdings (M,t)}\}]$ in the overall portfolio at time t, and {Short Portfolio (i,t)}, which includes all security holdings that are underweighted relative to $[(b_{Mit} + \theta_{Mit}) * \{\text{Benchmark Portfolio Holdings (M,t)}\}]$ in the overall portfolio at time t.¹⁷ All of the securities in the Short Portfolio will have negative weights to reflect the amount that they are underweighted. With this disaggregation, the manager's portfolio can be described as

$$(6) \text{ Portfolio Holdings (i,t)} = (b_{Mit} + \theta_{Mit}) * \{\text{Benchmark Portfolio Holdings (M,t)}\} + \{\text{Long Portfolio (i,t)}\} + \{\text{Short Portfolio (i,t)}\} + w_{it(\text{Cash})}$$

Following from (5), the Long Portfolio can be defined as

¹⁶ Such disaggregation can capture themes such as value / growth that are systematic or market-timing in nature. Unless such exposures are included in the benchmark portfolio, such exposure, in the long-run, should average out to zero to avoid systematic bias that would undermine the asset allocation decision.

¹⁷ Such a disaggregation would allow investors to evaluate whether a manager had the ability to identify investments that were both undervalued and overvalued. An example of this might be a Long / Short equity manager who relied on fundamental analysis to identify securities that were sufficiently mispriced relative to their estimated valuations. If it truly has a competitive edge, such an investment process ought to be able to identify attractive long *and* short opportunities over time. More generally, the appropriate disaggregation of the security selection portfolio should be targeted to the manager's particular style and strategy. This disaggregation should capture the important sources of incremental expected return and risk that are associated with the styles and strategies.

$$(7) \text{ \{Long Portfolio (i,t)\} = } \sum_k \text{ Max [0, } w_{kit} - (b_{Mit} + \theta_{Mit}) * w_{Mkt} \text{] (Security k)}$$

and the Short Portfolio can be defined as

$$(8) \text{ \{Short Portfolio (i,t)\} = } \sum_k \text{ Min [0, } w_{kit} - (b_{Mit} + \theta_{Mit}) * w_{Mkt} \text{] (Security k)}$$

The portfolio description in (6) reflects a two-step process whereby the manager first determines the desired exposure to the benchmark portfolio (including any desired market-timing activity captured by θ_{Mit}) and then implements security selection around this benchmark exposure. θ_{Mit} represents the manager's desired market timing decision, the deviation of β_{Mit} away from b_{Mit} , away from the prespecified asset allocation target exposure. By construction, the combined positions in the Long Portfolio and the Short Portfolio will create a portfolio with a zero beta, relative to the benchmark portfolio return as this combined portfolio is explicitly defined to only reflect the implementation of security selection decisions. Therefore, it follows that

$$(9) \beta_{Mt} (\text{Long Portfolio}) = - \beta_{Mt} (\text{Short Portfolio})$$

as both the Long Portfolio and the Short Portfolio are measured relative to $[(b_{Mit} + \theta_{Mit}) * \{\text{Benchmark Portfolio Holdings (M,t)\}]$ and θ_{Mit} will capture any change in the desired benchmark exposure of the manager's portfolio, explicitly or implicitly, from market timing.

It is noteworthy that, if an investment in an individual security is motivated by a desire to increase (or decrease) the systematic exposure of the portfolio, then the investment is a market-timing decision. For the same reason, if an investment in an individual security is motivated by return expectations that are dependent on the expected return of the

benchmark portfolio, then the investment is also a market-timing decision. Even if the manager is not intentionally making a market-timing decision, any deviations from b_{Mit} in the manager's portfolio will result in a deviation from the desired exposure to the benchmark portfolio from an asset allocation perspective, and therefore will result in changes in the expected return and risk characteristics of the overall portfolio. In most cases, the manager will have the ability to offset any unintended deviations from b_{Mit} through the use of benchmark-replicating derivatives. Therefore, any deviation by the manager from b_{Mit} should be evaluated as if it was an active management decision, and, specifically, a market-timing decision.

Of course, the manager's portfolio does not have to be created *a priori* in this two-step manner. The same decomposition can be carried out *ex post* to measure the separate contributions of market timing and security selection. It may be performed at any point in time by the manager by calculating β_{Mit} (or $b_{Mit} + \theta_{Mit}$) for the manager's portfolio and then calculating the Long Portfolio and Short Portfolio relative to $[(b_{Mit} + \theta_{Mit}) * \{\text{Benchmark Portfolio Holdings (M,t)}\}]$. The only information required is the then current portfolio holdings, the estimates of β_{Mkt} , and the known composition of the benchmark portfolio. While only observing the time series of returns, it may be impossible to separate the returns from market timing and security selection. However, the investment manager should always be able to make this separation.¹⁸

¹⁸ In the case of a hedge fund manager, the manager or the prime broker should be able to provide a wide range of disaggregation on a daily basis, capturing most of the major themes or strategies applicable to the manager's portfolio. At a minimum, the return and risk attribution associated with this disaggregation can be provided as part of the investment transparency available from the manager or the prime broker.

Active Management is Separable from the Passive Portfolio

Using the structure described above, it is straightforward to divide the manager's portfolio into a passive component and an active component

$$(10) \text{ Portfolio Holdings (i,t) = Passive Portfolio Holdings (i,t) + Active Portfolio Holdings (i,t).}$$

The composition of the Passive Portfolio Holdings represents the appropriate mix of the benchmark portfolio and cash to which the manager's performance will be compared. This passive portfolio must be a viable investment alternative, and its implementation should not depend on the skill-based investment decisions of active management. The composition of this passive portfolio is determined by the prespecified target exposures to both the benchmark portfolio and to cash which is consistent with the investor's overall asset allocation objective.¹⁹ Again, stating the obvious, there must be a *prespecified*, passive portfolio for all managers. As discussed below, for many managers (and especially hedge fund managers) this passive portfolio may simply be a 100 percent investment in cash.

The Passive Portfolio Holdings must utilize 100% of the investment capital allocated to the active manager. This requirement is necessary in order for there to be a fair comparison between the performance of the active manager's portfolio and the passive alternative. As this passive portfolio is specifically designed to be a feasible alternative to investing the allocated capital with the active manager, the only way the comparison will

¹⁹ Combining the passive portfolios of all of the managers employed by an investor (including any assets managed by the investor himself) should result in an overall passive portfolio that matches the investor's predetermined policy portfolio, resulting from the investor's asset allocation decision. See Sharpe (1988) for an excellent overview of the asset allocation decision process. Such a process can explicitly take into account the funding of liabilities and the risk characteristics of those liabilities will certainly impact the asset allocation decision, as demonstrated in Leibowitz and Henriksson (1988). All of the analysis in this paper is also applicable as part of an asset allocation process that includes liabilities.

be fair is if both the active manager and the passive alternative are assigned the same amount of investment capital. Therefore, the passive portfolio can be written as

$$(11) \text{ Passive Portfolio Holdings (i,t)} = b_{Mit} * \{\text{Benchmark Portfolio Holdings (M,t)}\} + (1 - b_{Mit}) \text{ (Cash)}$$

In many cases, the passive portfolio will not include any cash, as all of the capital will be invested in the benchmark portfolio. In this case, $b_{Mit} = 1$ and the passive portfolio will just be the benchmark portfolio. Alternatively, the passive portfolio may be entirely invested in cash, with $b_{Mit} = 0$, as the passive portfolio will have nothing invested in the benchmark portfolio.

Combining (3) and (11) results in

$$(12) \text{ Passive Portfolio Holdings (i,t)} = b_{Mit} * \sum_k W_{Mkt} \text{ (Security k)} + (1 - b_{Mit}) \text{ (Cash)}$$

The Active Portfolio Holdings can then be determined from (1) and (12), resulting in

$$(13) \text{ Active Portfolio Holdings (i,t)} = \sum_k \{(W_{kit} - b_{Mit} * W_{Mkt}) \text{ (Security k)}\} + W_{it} \text{ (\Delta Cash)}$$

where $W_{it} \text{ (\Delta Cash)}$ reflects the incremental cash requirements of the manager's desired market timing and security selection at time t.

The active portfolio will exclusively comprise all deviations from the prespecified Passive Portfolio Holdings. The active portfolio will include all investments required to implement market-timing and security-selection decisions and, as defined, all deviations from the prespecified passive portfolio can be classified as either market timing or as

security selection. Therefore, the separation of active management from the passive portfolio is simply a matter of subtracting the portfolio holdings that comprise the appropriate passive portfolio from the total portfolio holdings.

Following the same disaggregation used in (6), the active portfolio described in (13) can be thought of as

$$(14) \text{ Active Portfolio Holdings } (i,t) = \theta_{Mi} * \{ \text{Benchmark Portfolio Holdings } (M,t) \} + \\ \{ \text{Long Portfolio } (i,t) \} + \{ \text{Short Portfolio } (i,t) \} + \\ W_{it} (\Delta \text{ Cash})$$

The active portfolio includes all of the investments that are the result of market-timing or security-selection decisions and nothing else. The active portfolio reflects the complete separation of the skill-based active management decisions from the applicable underlying passive portfolio.

Active Management must be Self-Financing

The allocation of investment capital can be considered using the same framework. The cash holdings of the manager's overall portfolio will reflect the cash available after funding the Benchmark Portfolio Holdings exposure ($b_{Mit} + \theta_{Mit}$) and the Security Selection Portfolio. As the manager's portfolio must account for 100% of investment capital,²⁰ it must follow that

$$(15) \text{ Investment Capital (i,t)} = (b_{Mit} + \theta_{Mit}) + W_{it}(\text{Long Portfolio}) + W_{it}(\text{Short Portfolio}) + W_{it}(\text{Cash}) \\ = 1$$

where $W_{it}(\text{Long Portfolio})$ represents the cash required to fund the overweighted positions and $W_{it}(\text{Short Portfolio})$ represents the cash generated from the underweighted positions.

Following from (7), the proportion of investment capital required to implement the long portfolio, $W_{it}(\text{Long Portfolio})$, will be

$$(16) W_{it}(\text{Long Portfolio}) = \sum_k \text{Max} [0, w_{kit} - (b_{Mit} + \theta_{Mit}) * w_{Mkt}]$$

and following from (8), the proportion of investment capital generated from the short portfolio, $W_{it}(\text{Short Portfolio})$, will be

$$(17) W_{it}(\text{Short Portfolio}) = \sum_k \text{Min} [0, w_{kit} - (b_{Mit} + \theta_{Mit}) * w_{Mkt}]$$

²⁰ The exception, as discussed above, will be when the manager is not allocated any capital. This will require the entire portfolio to be self-financing. In that case, (15) will simply be

$$\text{Investment Capital (i,t)} = (b_{Mit} + \theta_{Mit}) + W_{it}(\text{Long Portfolio}) + W_{it}(\text{Short Portfolio}) + W_{it}(\text{Cash}) = 0.$$

By rearranging (15) the proportion of investment capital that is held as cash (or that is borrowed if negative) can be defined as

$$(18) w_{it}(\text{Cash}) = 1 - b_{Mit} - \theta_{Mit} - W_{it}(\text{Long Portfolio}) - W_{it}(\text{Short Portfolio})$$

The financing requirements of the Long Portfolio will always subtract from cash and the proceeds from the Short Portfolio will always add to cash. Market-timing activity can either require or generate incremental cash. Negative values for $w_{it}(\text{Cash})$ reflect the required borrowing necessary to implement the manager's desired overall portfolio, including the active management component, capturing all market-timing and security-selection decisions.

As discussed above, the required investment to create the Passive Portfolio Holdings must be 100% of the investment capital. The appropriate passive portfolio used in comparison with an active manager's portfolio performance must be allocated the same amount of investment capital as the manager's overall portfolio, which will be 100% of investment capital. Following from the definition of Passive Portfolio Holdings in (11), this allocation simply reflects the investment in the benchmark portfolio and any potential incremental cash

$$(19) \text{Passive Investment Capital (i,t)} = b_{Mit} + (1 - b_{Mit}) = 1.$$

Following from (14), the required investment to acquire the Active Portfolio Holdings will be

$$(20) \text{Active Investment Capital (i,t)} = \theta_{Mit} + W_{it}(\text{Long Portfolio}) + W_{it}(\text{Short Portfolio}) +$$

$$W_{it} (\Delta \text{ Cash})$$

where $w_{it} (\Delta \text{ Cash})$ reflects the incremental cash generated from or required to implement the desired market-timing and security-selection decisions.

The incremental cash requirements must reflect the fact that no investment capital is allocated to active management. This follows from (15) and (19)

$$(21) \text{ Active Investment Capital (i,t) = Investment Capital (i,t) -} \\ \text{Passive Investment Capital (i,t) = } 1 - 1 = 0.$$

While obvious, this is a powerful result. Active management, as measured by the deviation of the portfolio holdings from those of the passive portfolio, must be completely self-financing as it is not allocated any investment capital. Therefore, in the aggregate, all active management decisions must be implemented with zero investment capital.²¹ Any cash required to implement active management investments will require financing (either explicit or implicit), with the accompanying financing costs. Any cash generated from the active management decisions will earn the prevailing return on cash.

The fact that active management must be self-financing follows directly from the separation of active management from the passive portfolio, as captured in (14). This separation allows any active management decisions to be implemented without

²¹ As shown below, all active management decisions can be evaluated based on their differential return, relative to the available riskless return or financing rate. From this perspective all active management decisions can be evaluated from the incremental return from taking risk. Such a differential return will also capture the economics of a self-financing or zero-investment strategy as each investment must be considered net of the implicit borrowing cost of financing the desired investment. The Sharpe Ratio is the most well-known example of a performance measurement tool which is based on a differential return that represents such a zero-investment strategy. Sharpe (1994) clearly lays out the logic for using differential returns for performance measurement.

consideration of the underlying allocation of investment capital or the underlying passive portfolio. Thereby, from an asset allocation perspective, any active management portfolio can be implemented as an overlay strategy on top of any passive portfolio. Regardless of differences in the underlying passive portfolio or the actual amount of capital allocated to a manager, active management decisions can be executed to achieve the same active management portfolio.

Given the self-financing requirement, the incremental cash associated with active management must exactly offset the cash requirements of the active management investments, resulting in

$$(22) W_{it}(\Delta \text{ Cash}) = -\theta_{Mit} - W_{it}(\text{Long Portfolio}) - W_{it}(\text{Short Portfolio})$$

θ_{Mit} can be either negative or positive; with the result that market timing can either generate incremental cash or require financing. $W_{it}(\text{Long Portfolio})$ will never be negative and $W_{it}(\text{Short Portfolio})$ will never be positive. Therefore, the incremental cash from security selection can be positive or negative, depending on the relative values of the betas of the securities in the Long Portfolio and the Short Portfolio.²²

As described in (18), $W_{it}(\text{Cash})$ combines the cash component of the passive portfolio ($1 - \beta_{Mit}$) with the incremental cash generated from active management ($-\theta_{Mit} - W_{it}(\text{Long Portfolio}) - W_{it}(\text{Short Portfolio})$). Positive incremental cash can be assumed to earn R_{ft} and negative

²² From (10), it is required that $\beta_{Mt}(\text{Long Portfolio}) = -\beta_{Mt}(\text{Short Portfolio})$. However, there is no requirement that $W_{it}(\text{Long Portfolio}) = -W_{it}(\text{Short Portfolio})$. Consider the simple case where the Long Portfolio consists of only one security, with a beta of 1.0 with the benchmark portfolio and the Short Portfolio consists of only one security, with a beta of 0.5 with the benchmark portfolio. $\beta_{Mt}(\text{Long Portfolio})$ will be offset by $\beta_{Mt}(\text{Short Portfolio})$ if the (negative) size of the Short Portfolio is twice as large as the (positive) size of the Long Portfolio. If the investment required to implement the Long Portfolio was \$1 and proceeds from short sale of the Short Portfolio was \$2, the combined beta of the Security Selection Portfolio will equal zero, but the Security Selection Portfolio will generate \$1 in cash from the extra short sale proceeds.

incremental cash will result in required borrowing, at the appropriate financing cost. If the negative incremental cash is less than the amount of cash in the passive portfolio ($100\% - b_{Mit}$), then this borrowing comes from the cash in the passive portfolio, with an implicit financing cost of R_{ft} since R_{ft} reflects the opportunity cost of the incremental investments. Cash requirements beyond this amount will require actual borrowing from a counterparty.

In practice, the counterparties providing the financing required to implement the active management investments may require collateral beyond that available from the Active Portfolio Holdings. This may require an allocation of cash to what would otherwise be self-financing investments. However, through the use of derivatives or other benchmark-related holdings, it is possible, as will be demonstrated in the next section, to separate the cash requirements for collateral from the risk and return characteristics desired from the asset allocation decision. In effect, investment capital allocated to passive investments, consistent with the policy portfolio, can be freed up for use as collateral for the active portfolio, without impacting the overall asset allocation. Therefore, from an economic perspective, collateral requirements need not violate the requirement that active management be self-financing.²³ Any active management portfolio can be implemented as if it were an overlay strategy on top of any passive portfolio, including the cash requirements of collateral, from an asset allocation perspective. Likewise, any active management decisions can be implemented separate from the underlying allocation of investment capital due to their self-financing characteristic.

²³ Any incremental costs associated with the active manager's collateral requirements can be thought of as financing costs and should be charged to the active manager.

Active Management is Portable

The simple result that active management must be self-financing takes on great significance because it allows any active management portfolio to be combined with any passive portfolio. Because of the self-financing requirement, active management can always be thought of as being implemented through an overlay process, relative to any passive portfolio. In practice, this allows the active management decision to be completely separated from the policy portfolio reflected in the asset allocation decision. Active management can, and should, be evaluated as if it is being implemented on top of the policy portfolio which, by definition, will generate exclusively systematic returns from “beta” exposure. From this perspective, active management should generate returns exclusively from “alpha.” By virtue of the separability of the passive and active returns within the descriptive framework, the “alpha” achievable from active management will be portable as it can be combined with any passive portfolio (capturing the desired systematic return, or “beta”) without having any impact on the active manager’s investment decisions.

This portability can be easily accomplished for any manager’ portfolio. The underlying passive portfolio can be transformed to another passive portfolio with no incremental cash requirements and can usually be carried out with the use of derivatives. Consider an investor who wants to invest with an active manager who uses the passive portfolio shown in (12), including benchmark portfolio M. From (14), the active portfolio holdings for the manager are

$$(23) \text{ Active Portfolio Holdings } (i,t) = \theta_{Mit} * \sum_k w_{Mkt} (\text{Security } k) + \sum_k \text{Max } [0, w_{kit} - (b_{Mit} + \theta_{Mit}) * w_{Mkt}] (\text{Security } k) +$$

$$\sum_k \text{Min} [0, w_{kit} - (b_{Mit} + \theta_{Mit}) * w_{Mkt}] \text{ (Security } k) -$$

$$(\theta_{Mit} + w_{it} \text{ (Long Portfolio)} + w_{it} \text{ (Short Portfolio)}) \text{ (Cash)}$$

However, from an asset allocation perspective, the investor prefers the following passive portfolio

$$(24) \text{ New Passive Portfolio Holdings (i,t)} = b_{Pit} * \{\text{Benchmark Portfolio Holdings (P,t)}\}$$

$$+ (1 - b_{Pit}) \text{ (Cash)}$$

where $\{\text{Benchmark Portfolio Holdings (P,t)}\}$ represents the exact security composition of the benchmark portfolio P, at time t, and represents a passive, investable portfolio that comprises the following holdings

$$(25) \{\text{Benchmark Portfolio Holdings (P,t)}\} = \sum_k w_{Pkt} \text{ (Security } k)$$

where w_{Pkt} is the proportion of the benchmark portfolio invested in Security k and it is required that $\sum_k w_{Pkt} = 1$.

The investor can transform the active manager's underlying passive portfolio by implementing the following investments

$$(26) \{\Delta \text{ Passive Portfolio Holdings (i,t)}\} = b_{Pit} * \sum_k w_{Pkt} \text{ (Security } k) -$$

$$b_{Mit} * \sum_k w_{Mkt} \text{ (Security } k) + (b_{Mit} - b_{Pit}) \text{ (Cash)}$$

$\{\Delta \text{ Passive Portfolio Holdings (i,t)}\}$ effectively transforms one passive portfolio into an alternative passive portfolio. The investment requires zero net cash as the cash required to

transform the benchmark portfolio exposure, $(b_{Pit} - b_{Mit})$, is offset by the change in the desired passive cash $(b_{Mit} - b_{Pit})$.²⁴

Combining (6) and (26) results in a new portfolio that exactly preserves the entire active management portfolio, while changing the underlying passive portfolio from one including asset class M to one including asset class P. The result is

$$\begin{aligned}
 (27) \text{ Portfolio Holdings } (i,t) = & b_{Pit} * \sum_k w_{Pkt} \text{ (Security } k) + (1 - b_{Pit}) \text{ (Cash)} + \\
 & \theta_{Mit} * \sum_k w_{Mkt} \text{ (Security } k) + \\
 & \sum_k \text{Max } [0, w_{kit} - (b_{Mit} + \theta_{Mit}) * w_{Mkt}] \text{ (Security } k) + \\
 & \sum_k \text{Min } [0, w_{kit} - (b_{Mit} + \theta_{Mit}) * w_{Mkt}] \text{ (Security } k) - \\
 & (\theta_{Mit} + w_{it}(\text{Long Portfolio}) + w_{it}(\text{Short Portfolio})) \text{ (Cash)}
 \end{aligned}$$

The passive portfolio included in (6) is neutralized by an offsetting short exposure included in (26) and is replaced by the desired new passive portfolio, which comprises

$$(28) \text{ New Passive Portfolio Holdings } (i,t) = b_{Pit} * \sum_k w_{Pkt} \text{ (Security } k) + (1 - b_{Pit}) \text{ (Cash)}$$

This new passive portfolio results from the long exposure in (26). The new overall portfolio holdings combine positions corresponding to the new passive portfolio plus an active portfolio which is exactly the same as the composition of the active portfolio prior

²⁴ This must be true as the capital assigned to the passive portfolio will not change and, as discussed above, the passive portfolio will be assigned all of the capital assigned to the manager. In the case where neither benchmark portfolio includes any cash, (26) will simply be

$$\{\Delta \text{ Passive Portfolio Holdings } (i,t)\} = \sum_k w_{Pkt} \text{ (Security } k) - \sum_k w_{Mkt} \text{ (Security } k)$$

since $b_{Pit} = b_{Mit} = 1$. In this case, the long position in benchmark portfolio P will be financed by the proceeds from the short position in benchmark portfolio M.

to the transformation of the underlying passive portfolio. Before and after the passive portfolio transformation, the active portfolio remains

$$\begin{aligned}
 (29) \text{ Active Portfolio Holdings (i,t)} &= \theta_{Mit} * \sum_k W_{Mkt} \text{ (Security k)} + \\
 &\sum_k \text{Max} [0, w_{kit} - (b_{Mit} + \theta_{Mit}) * W_{Mkt}] \text{ (Security k)} + \\
 &\sum_k \text{Min} [0, w_{kit} - (b_{Mit} + \theta_{Mit}) * W_{Mkt}] \text{ (Security k)} - \\
 &(\theta_{Mit} + W_{it}(\text{Long Portfolio}) + W_{it}(\text{Short Portfolio})) \text{ (Cash)}
 \end{aligned}$$

Despite the different underlying passive portfolios, the Active Portfolio Holdings are exactly the same in (23) and (29).

In many cases, the portfolio described in (26) can be created directly by combining long and short positions in the underlying securities and executing the transaction as a basket trade. For most commonly used asset class benchmarks, the combined portfolio described in (26) can be easily created through the use of swaps, futures, or other index-replicating securities, with minimal incremental costs.²⁵ Therefore, the investment universe that provides the source of alpha from active management does not need to be restricted to an asset class embedded in the policy portfolio from an asset allocation perspective. The alpha from active management is described as portable because it can be combined with

²⁵ By incremental costs, we are referring to transaction costs or other such investment frictions that will result in investment performance that deviates from the performance of a direct investment in the benchmark portfolio. Such “costs” may actually be negative. This would be the case, for example, if the aggregate bid/ask spread was greater for an investment in the individual securities comprising the benchmark than for the replicating derivative. It would also be the case if index futures contracts were theoretically mispriced such that the index could be purchased more cheaply through the combination of futures contracts and cash than by direct investments in the securities comprising the index. Possible losses from the exposure to the benchmark portfolio should not be included in the potential costs associated with active management since such losses would be simply due to the performance of the desired passive portfolio. Therefore, any such loss or gain should be evaluated from the perspective of the policy portfolio decision, separate from the evaluation of the active management decision.

any asset class investment. Likewise, an active manager should be free to pursue attractive investment opportunities without asset allocation-related restrictions. The only restrictions should be directly related to the active management investment process, taking into account such considerations as the active manager's areas of skill-based expertise and the risk management and monitoring capabilities of the active management vehicle.

Cash that is allocated to an active manager can also be transformed in the same way to fit into the investor's desired overall asset allocation. This will include any cash required for purposes of collateral. Any allocation of cash can be transformed to capture the systematic expected return and risk characteristics of almost any asset class in the policy portfolio. This transformation is accomplished through the substitution of cash for asset class M in (26), resulting in the same payoff available from futures, forwards, or swap contracts. The return and risk characteristics of such a cash allocation can be transformed to fit into the desired asset allocation with the following investment

$$(30) \{\Delta \text{ Passive Portfolio Holdings (i,t)}\} = b_{Pit} * \sum_k (W_{Pkt} \text{ (Security k)} - (\text{Cash}))$$

In this way, the allocation of cash is also separable from the asset allocation decision. Any cash requirements can be offset through the use of overlay transactions such as described in (30). In most cases, the desired transformation for asset allocation purposes can be executed without the involvement of the active manager. The transformation can simply be carried out as an overlay to the capital allocation. As will be demonstrated below, it follows that the applicable passive portfolio for any active manager charging performance-related fees should be a 100% investment in cash.

The Concept of Leverage is Arbitrary

The simple result that active management must be self-financing also makes the concept of leverage arbitrary. This is due to the fact that the assignment of capital is, in itself, an arbitrary decision with respect to active management. Just as the active management decision can be completely separated from the underlying passive portfolio, it can also be separated from the decision as to how much capital is allocated to the passive portfolio. The amount of capital allocated to active management will certainly impact the return and risk characteristics of an active manager's portfolio when the returns and risks are measured per unit of capital. However, such measures are only relevant to the extent that they impact the investor's overall asset allocation. As all active management must be self-financing, any decisions regarding active management can be implemented as an overlay on the desired asset allocation, assuming no net allocation of capital. All that matters is the expected return and risk characteristics associated with the investment in active management and the impact of those return and risk characteristics on the overall portfolio.

The fact that the allocation of capital is arbitrary is demonstrated by the example shown in Table I. In Table I, the same overall set of investments is undertaken, with three different allocations of \$1000 of investment capital between an Index Manager and an Active Manager. In all three cases, the investor's desired policy portfolio comprises an investment of \$900 in the Benchmark Portfolio and an investment of \$100 in Cash. In the aggregate, the Passive Portfolios of the Index Manager and the Active Manager must equal the policy portfolio. In all three cases, \$100 in cash is also required to implement the desired Active Management Portfolio. In all three cases, both borrowing and lending (investing in Cash) are assumed to be riskless.

In the first case, the capital is simply allocated as required, with \$900 allocated to the Index Manager and \$100 allocated to the Active Manager. All of the allocation to the Index Manager is an allocation to the Passive Portfolio and all \$900 is invested in the Benchmark Portfolio. From the perspective of (11), $b_{Mit} = 1$ for the Index Manager. The Index Manager has an expected return of \$72 or 8% from the investment in the Benchmark Portfolio. The Index Manager has no investments in an Active Portfolio.

All of the money allocated to the Active Manager should also be thought of as an allocation to the Passive Portfolio, with all \$100 allocated to Cash, earning \$5 or 5%. From the perspective of (11), $b_{Mit} = 0$ for the Active Manager. The Active Portfolio of the Active Manager has an expected return of \$20, but requires \$100 to implement the active decisions which must be financed (implicitly from the Cash in the Passive Portfolio), with a financing cost of \$5 or 5%. This borrowing of \$100 is explicitly allocated to the Active Portfolio. Combining the Passive Portfolio of the Active Manager with the Active Portfolio and its required financing, results in a \$20 expected return for the Active Manager ($\$5 + \$20 - \$5$), an expected return of 20% on the allocated capital of \$100.

In combination, the Passive Portfolios of the Index Manager and the Active Manager are allocated \$1000, or 100% of all investment capital, and the combined investments exactly match the policy portfolio with \$900 invested in the Benchmark Portfolio and \$100 invested in Cash. This results in an expected return of \$77, or 7.7% on the combined Passive Portfolios. No capital is allocated to the Active Portfolio of the Active Manager and the Index Manager has no active investments. Instead, all of the investments in the Active Portfolio must be financed, which in this case requires \$100. Net of financing costs, the Active Portfolio will have an expected return of \$15. Together, the combined expected return from the Passive Portfolios and the Active Portfolio is \$92 ($\$77 + \15), or

9.2% on the total investment capital of \$1000. This is also the combined expected return from the allocations to the Index Manager and the Active Manager (\$72 + \$20).

In the second case, all of the capital is allocated to the Index Manager and none is allocated to the Active Manager. All of the allocation to the Index Manager is an allocation to the Passive Portfolio, with \$900 invested in the Benchmark Portfolio and \$100 invested in Cash. From the perspective of (11), $b_{Mit} = 0.9$ for the Index Manager. This portfolio exactly matches the entire policy portfolio of the investor. The Index Manager has an expected return of \$77 or 7.7% from the investments in the Benchmark Portfolio and Cash. Again, the Index Manager has no investments in an Active Portfolio.

No capital is allocated to the Active Manager which also requires that no capital is allocated to the Passive Portfolio of the Active Manager. Again, the Active Portfolio of the Active Manager has an expected return of \$20, but requires \$100 to implement the active decisions which must be explicitly financed, with the borrowing allocated to the Active Portfolio, as the Active Manager is not allocated any capital. The required borrowing results in a financing cost of \$5 or 5%. As there is no Passive Portfolio, the expected return for the Active Manager will simply come from his Active Portfolio investments, less the required financing cost, resulting in a \$15 expected return for the Active Manager (\$20 - \$5), with no allocated capital.²⁶

In this case, all of the investment capital is invested in the Passive Portfolio of the Index Manager, with an allocation of \$1000 and investments that exactly match the policy portfolio with \$900 invested in the Benchmark Portfolio and \$100 invested in Cash. Therefore, the Active Manager will not have a Passive Portfolio. All of the investments in

²⁶ Of course, the investor will still be responsible for any losses that result from the Active Portfolio and the related financing costs. In effect, the capital allocated to the Index Manager will serve as collateral for the investments in the active portfolio.

the Active Portfolio must be financed. Despite the different allocation of capital, the overall portfolio has exactly the same combined expected return: \$92 (\$77 + \$15) or 9.2%.

In the third case, \$800 is allocated to the Index Manager and \$200 is allocated to the Active Manager. All of the allocation to the Index Manager is again an allocation to the Passive Portfolio, but in this case the \$800 allocation is not sufficient to satisfy the required \$900 investment in the Benchmark Portfolio that the investor desires from the perspective of the policy portfolio. The Index Manager needs to borrow \$100, with a financing cost of \$5 or 5%, in order to make the desired investment in the Benchmark Portfolio. From the perspective of (11), $b_{Mit} = 1.125$ for the Index Manager. In this case, the Passive Portfolio for the Index Manager will include both the \$900 investment in the Benchmark Portfolio and the \$100 in borrowing (a negative investment in Cash). The Passive Portfolio of the Index Manager, which represents his entire portfolio, is a leveraged investment in the Benchmark Portfolio and has an expected return of \$67 or 8.375% on a net investment of \$800.

All of the money allocated to the Active Manager should also be thought of as an allocation to his Passive Portfolio, with all \$200 allocated to Cash, earning \$10 or 5%. From the perspective of (11), $b_{Mit} = 0$ for the Active Manager. The Active Portfolio of the Active Manager again has the same expected return of \$20 and the same active investment requirement of \$100 to implement the active decisions. This active investment requirement must be financed (implicitly from the Cash in the Passive Portfolio), with a financing cost of \$5 or 5%, and this borrowing is explicitly allocated to the Active Portfolio. Combining the Active Manager's Passive and Active Portfolios and his required financing cost, results in a \$25 expected return for the Active Manager (\$10 + \$20 - \$5), an expected return of 12.5% on the allocated capital of \$200.

In combination, the Passive Portfolios of the Index Manager and the Active Manager are allocated \$1000 and the combined investments exactly match the policy portfolio with \$900 invested in the Benchmark Portfolio and a net investment of \$100 in Cash. The Cash investment results from combining the \$200 from the Active Manager's Passive Portfolio with the \$100 in borrowing in the Index Manager's Passive Portfolio. This results in the same expected return of \$77, or 7.7% on the combined Passive Portfolios. As in the first two cases, no money is allocated to the Active Portfolio of the Active Manager and the Index Manager has no active investments. All of the investments in the Active Portfolio must be financed, which again requires \$100. Net of financing costs, the Active Portfolio will have an expected return of \$15. Together, the combined expected return from the Passive Portfolios and the Active Portfolio is \$92 (\$77 + \$15), or 9.2% on the total investment capital of \$1000. This is also the combined expected return from the allocations to the Index Manager and the Active Manager (\$67 + \$25).

Up to this point, all of the analysis has focused on the expected returns of the portfolios. In the same way, the standard deviation of the returns for the overall portfolio must also be the same for all three cases considered in Table I. When the returns from the Benchmark Portfolio and the Active Portfolio are assumed to be uncorrelated ($\rho = 0$), the standard deviation of the overall portfolio will always be \$92.2 or 9.22%. Regardless of the allocation of capital, the standard deviation for the overall portfolio will only depend on the standard deviation of returns from the Benchmark Portfolio and the Active Portfolio and the correlation between the returns of the two sets of investments, assuming that both borrowing and investing in Cash are riskless.

In all three cases, the investor's overall expected return and standard deviation are not impacted by the allocation of capital as long as the allocation does not impact the investment decisions.

This result is just a variation on the seminal work of Modigliani and Miller (1958, 1959) on the capital structure decision of the firm. As long as there is no impact on the aggregate cash flows to the investor, the allocation of capital amongst the individual investments will be irrelevant. In this case, this will be true as long as the overall investment portfolio does not change, the respective borrowing and lending rates are the same, and there are no other differential costs.²⁷

This result does not mean that the leverage used by a manager, given an assigned amount of investment capital, does not matter. Given a fixed capital base, the amount of leverage will influence the magnitude of the expected return opportunity and the magnitude of the risks associated with the investments. In this case, the amount of leverage will be directly linked to the size of the overall positions and it is the size of the positions that will determine the expected return and risk characteristics that will impact the overall investment portfolio. However, these return and risk characteristics are determined by the portfolio of active investments, independent of the amount of capital assigned to the investments. The amount of leverage is simply the result of taking the size of the portfolio of active investments and dividing by the allocated amount of capital. Given an allocation of capital, the optimal leverage for a manager will simply be the amount required to fund the optimal portfolio of investments. The decision with respect to how

²⁷ One possible differential cost would be management fees. If a portion of the management fee is a fixed percentage of allocated capital, then everything else equal, an investor would always desire to allocate the least capital possible. In practice, such a complication can easily be resolved. It is not uncommon with managed accounts for the manager to let the investor choose, with the agreement of the prime broker, the amount of capital to be allocated to the account. In this case, the position sizes and the management fee are based on an agreed-upon notional amount instead of the actual allocated capital.

much active management should not depend on the amount of capital allocated, but rather upon the incremental impact of the active management on the expected return and risk characteristics of the overall investment portfolio.

Return and Risk Attribution

Up to this point, the analysis has only focused on the portfolio holdings of the manager to achieve the desired portfolio disaggregation used to separate the sources of active management from their underlying passive portfolio. It is straightforward to use the same portfolio disaggregation to evaluate a manager's returns. Given that returns will always be additive, the return on a portfolio equals the weighted average of the returns of the securities or sub portfolios that make up the portfolio. Following from (1), the return on a manager's portfolio can be written as

$$(31) R_{it} = \sum_k (w_{kit} * R_{kt}) + w_{it(Cash)} * R_{ft}$$

where R_{it} is the one-period return per dollar generated by manager i , R_{ft} is the one-period return per dollar on riskless securities, and R_{kt} is the one-period return per dollar on Security k , with the return period beginning at time t . Following from (6), these security returns can be grouped to reflect the passive and active management components and shown as

$$(32) R_{it} = (b_{Mit} + \theta_{Mit}) * R_{Mt} + w_{it(Long\ Portfolio)} * R_{it(Long\ Portfolio)} + w_{it(Short\ Portfolio)} * R_{it(Short\ Portfolio)} + w_{it(Cash)} * R_{ft}$$

where R_{Mt} is the one-period return on the benchmark portfolio. $R_{it(Long\ Portfolio,t)}$ is the return, per dollar of investment, earned on the Long Portfolio and following from (7) can be expressed as

$$(33) R_{it(Long\ Portfolio)} = \sum_k \{ \text{Max} [0, w_{kit} - (b_{Mit} + \theta_{Mit}) * w_{Mkt}] * R_{kt} \} / w_{it(Long\ Portfolio)}$$

Likewise, R_{it} (Short Portfolio) is the return, per dollar of investment, earned on the Short Portfolio and following from (8) can be expressed as

$$(34) R_{it} \text{ (Short Portfolio)} = \frac{\sum_k \{ \text{Min} [0, w_{kit} - (b_{Mit} + \theta_{Mit}) * w_{Mkt}] * R_{kt} \}}{w_{it} \text{ (Short Portfolio)}}$$

The portfolio return shown in (32) can be rearranged as follows

$$(35) R_{it} = R_{ft} + (b_{Mit} + \theta_{Mit}) * (R_{Mt} - R_{ft}) + \{ w_{it} \text{ (Long Portfolio)} * [R_{it} \text{ (Long Portfolio)} - R_{ft}] + w_{it} \text{ (Short Portfolio)} * [R_{it} \text{ (Short Portfolio)} - R_{ft}] \}$$

Looked at in this way, all of the components of the return are expressed as returns net of the riskless rate, or the financing rate, and can be thought of as the return from taking risk.

Following from (11), the return on the passive portfolio can be written as

$$(36) R_{it} \text{ (passive)} = b_{Mit} * R_{Mt} + (1 - b_{Mit}) * R_{ft}$$

where $R_{it} \text{ (passive)}$ is the one-period return per dollar on the prespecified, passive portfolio for manager i . By investing b_{Mit} in the benchmark portfolio and the remaining investment capital $(1 - b_{Mit})$ in cash, an investor can replicate the passive portfolio used to evaluate the active manager. Therefore, the passive portfolio is always a viable investment alternative to the active manager.

In the same spirit as (35), the passive return can also be thought of as combining the return that could be earned from cash on the entire investment capital plus the excess return earned on the target exposure to the benchmark portfolio. Rearranging (36) results in

$$(37) R_{it} (\text{passive}) = R_{ft} + b_{Mit} * (R_{Mt} - R_{ft})$$

Following from (37), rearranging (35) to isolate the returns due to skill-based active management from the returns available from the passive portfolio results in

$$(38) R_{it} = \{R_{ft} + b_{Mit} * (R_{Mt} - R_{ft})\} + \{ \theta_{Mit} * (R_{Mt} - R_{ft}) + \\ W_{it} (\text{Long Portfolio}) * [R_{it} (\text{Long Portfolio}) - R_{ft}] + \\ W_{it} (\text{Short Portfolio}) * [R_{it} (\text{Short Portfolio}) - R_{ft}] \}$$

The first term in brackets in (38) is the return available from the passive portfolio shown in (11). The second term in brackets in (38) represents the return from active management corresponding to the portfolio specified in (14). Since the return to active management captures all deviations between the portfolio's total return and the appropriate passive portfolio return, the return from active management is

$$(39) R_{it} (\text{active}) = \theta_{Mit} * (R_{Mt} - R_{ft}) + \{ W_{it} (\text{Long Portfolio}) * [R_{it} (\text{Long Portfolio}) - R_{ft}] + \\ W_{it} (\text{Short Portfolio}) * [R_{it} (\text{Short Portfolio}) - R_{ft}] \}$$

where the return is divided into the component that comes from market timing ($\theta_{Mit} * (R_{Mt} - R_{ft})$) and the component that comes from security selection ($W_{it} (\text{Long Portfolio}) * [R_{it} (\text{Long Portfolio}) - R_{ft}] + W_{it} (\text{Short Portfolio}) * [R_{it} (\text{Short Portfolio}) - R_{ft}]$). By evaluating the performance of the active manager relative to the prespecified passive portfolio, it is possible to isolate the returns that are the result of skill-based, active decisions.

All of the components of active returns can be expressed as differential returns. By looking at each source of return relative to R_{ft} , this framework highlights the fact that the active portfolio investments are not allocated any investment capital. The returns from

active management must include the cost of financing the positions, whether that financing requires explicit borrowing or is implicitly financed through the use of portfolio cash. As the returns from active management come from taking risk, it is appropriate to consider the returns incremental to R_{ft} , the return that is available without taking risk. This framework also highlights the fact that short positions will generate cash from the proceeds of the short sales (or the reduced positions relative to the passive portfolio) and that cash will earn R_{ft} .

In performance evaluation, it will always be desirable to examine the appropriate disaggregated returns. For example, the performance of market timing activity can be evaluated from

$$(40) R_{it} (\text{Market Timing}) = \theta_{Mit} * (R_{Mt} - R_{ft})$$

and the performance of security selection can be evaluated from

$$(41) R_{it} (\text{Security Selection}) = W_{it} (\text{Long Portfolio}) * [R_{it} (\text{Long Portfolio}) - R_{ft}] + \\ W_{it} (\text{Short Portfolio}) * [R_{it} (\text{Short Portfolio}) - R_{ft}]$$

From the portfolio holdings, the investment manager or the prime broker should always be able to provide disaggregated returns that capture all of the important sources of incremental expected return and risk that are associated with the relevant investment styles and strategies. Even if such returns are only available with some time delay, they should be able to provide substantial information that can aid an investor in the manager selection process and the ongoing risk monitoring process. And, in most cases, this information can be made available without having to give up the identity of the exact security positions, either explicitly or by inference.

All of the previous analysis follows from the portfolio holdings of an investment manager. It is important to realize that the resulting specification of returns is quite limited in what it does and what it assumes. What it does is disaggregate one-period returns relative to a predetermined benchmark portfolio. To accomplish this, it is not necessary to make any assumptions regarding the appropriate equilibrium pricing model for securities or the time-series properties of the returns. As long as b_{Mit} is known, it will be possible to separate returns from active management from those available from passive management. This just requires that the appropriate benchmark is established in advance, which should always be the case.

If it is also possible to identify θ_{Mit} , then it will also be possible to separate returns generated by security selection from returns generated by market timing. While θ_{Mit} will not typically be known to investors in advance, it can always be calculated from the portfolio holdings at time t . Therefore, θ_{Mit} should certainly be known by the manager and can be easily calculated by the prime broker from the weighted-average estimates of β_{Mkt} for the securities in the portfolio.

Once θ_{Mit} has been determined, it is straightforward to calculate the returns attributable to security selection. Using the portfolio holdings, it will also be possible to further disaggregate the returns to better understand the sources of return and sources of risk in the active portfolio.

In general, this framework can be used to provide much of the desired return and risk attribution for any manager, including hedge fund managers. By disaggregating the portfolio to focus on the applicable themes, strategies, or risks for each hedge fund manager, investors will be able to obtain important insights into the sources of returns and the accompanying volatility that are generated by the manager's investment decisions. In

most cases, managers or prime brokers should be able to generate a daily time series of such returns, using no more information than should be required by the manager for effective risk management.²⁸

All this said, unfortunately, it will usually be quite difficult to accurately evaluate the statistical significance of most of the results. This is due to the inherent lack of stationarity in the underlying return generating process and the relatively short return histories for most hedge funds. However, this does not mean that the risk and return attribution is not worth the effort; it just means that any analysis based on historical returns should be considered with a good dose of skepticism. The analysis may not provide a definitive answer, but it should provide a great source of questions. Given this, the value of focused disaggregation of the historical returns will be especially important.

Investment decisions are going to be made regardless of the statistical significance of the returns. In such a setting, the manager selection process must depend on the best possible understanding of what goes into the active management decisions of a manager within each strategy. By combining the information from the disaggregated returns, and other risk-related information available from transparency, with comparable data from other

²⁸ In most cases, the exact portfolio holdings may change over the course of a trading day. In such cases, complete separation of market timing and security selection will not be possible with daily returns. If the daily returns are calculated from the portfolio holdings at the beginning of each day, the disaggregated return attribution described above will not exactly aggregate into the overall portfolio returns when the trading interval is shorter than the return interval. How much the portfolio returns differ will depend on how actively the manager trades the portfolio and how much the individual returns fluctuate during the trading day.

It is unlikely that this discrepancy can be resolved through the use of returns that are generated more frequently than daily, given the complications in obtaining contemporaneous security prices. Instead, the return discrepancy should be viewed as just another source of return disaggregation, potentially capturing the gains and losses from short-term trading activity and the potential transaction costs associated with executing the desired changes in the active management portfolio.

managers in the same strategy, it should be possible to make well-informed, qualitative judgments with respect to active managers and strategies.

Fees Make Hedge Funds Inefficient Sources of Passive Returns

Up to this point, the analysis of returns has not taken into account the fees charged by managers. Hedge funds typically charge a fixed percentage management fee, based on the assets under management at the beginning of each month, and a performance fee that is settled annually and pays a fixed percentage of the fund's profit, or profit above the return on cash or some other absolute threshold. As shown in (38), the profit in excess of the return on cash is simply the excess return from the benchmark portfolio plus the return from active management. As long as the sum of these returns has been profitable, the performance fee will simply accrue as a fixed percent of the incremental performance. In general, the performance fee will depend on the cumulative return of the manager for the year and possibly any cumulative losses from previous years if a "high-water mark" is applied. Using the framework outlined above, the two forms of fees are just another source of return disaggregation. For example, (38) can be modified to take into account the fees charged by any manager as follows

$$\begin{aligned}
 (42) \ R_{it} = & \{R_{ft} + b_{Mit} * (R_{Mt} - R_{ft})\} + \{ \theta_{Mit} * (R_{Mt} - R_{ft}) + \\
 & W_{it} \text{ (Long Portfolio)} * [R_{it} \text{ (Long Portfolio)} - R_{ft}] + \\
 & W_{it} \text{ (Short Portfolio)} * [R_{it} \text{ (Short Portfolio)} - R_{ft}] \} - \\
 & \text{Management Fee (i,t) - Performance Fee (i,t)}
 \end{aligned}$$

For a given starting point, both forms of fees can easily be calculated on a daily basis by the manager or the prime broker.²⁹

²⁹ The fee structure can obviously be modeled much more precisely, but this simple description is sufficient for our purposes. In addition, we are ignoring other expenses that may be charged to the fund by the manager. Such expenses can be important and will be quite difficult for the prime broker to calculate, except when they are actually charged to the fund. Such expenses will certainly impact the calculation of performance fees. However, they are most important for their aggregate impact on performance and fees.

As b_{Mit} is the prespecified asset allocation target value of β_{Mit} for manager i , it should be equal to the long-term average exposure to the systematic risk of the benchmark portfolio. As this source of return and risk can easily be separated from the active management portfolio, it will only make sense for the desired value of b_{Mit} to be something different from zero for a hedge fund manager if that manager can offer the passive portfolio return in a way that is competitive with index funds that are specifically structured to generate the passive returns in a cost effective manner. In many cases, the actual cost differential between hedge fund managers and their index fund counterparts will exceed one percent per annum, even before taking into account management fees. As the index funds (or derivative alternatives) also offer almost perfect transparency and flexible liquidity, the only way a hedge fund can ever be a competitive source of passive returns is if it can charge lower fees for generating the passive returns.

For hedge funds, the management fee will almost always be one percent or greater, while the management fee for index funds will always be substantially less than one percent and in some cases will be as small as couple of basis points. In addition, index funds do not charge a performance fee. Therefore, the only way that a hedge fund can ever charge lower or equal fees will be if the performance fee that is attributable to the passive return is sufficiently negative that it will offset all of the other cost advantages of index funds.

In the aggregate, hedge fund performance fees can never be negative. Therefore, the performance fee attributable to the passive return can only be negative to the extent that it is offset by the performance fee attributable to active management. With a typical performance fee for hedge funds of twenty percent, it will take, at a minimum, an annual loss in excess of five percent in the passive portfolio, plus an accompanying gain in

Except in those situations where the expenses are explicitly tied to a particular source of disaggregated returns, they should be treated as one additional disaggregated cash flow.

excess of five percent from active management, to offset the cost advantages of an index fund. While this outcome is possible, the annual cost differential must be overcome on average over the timeframe used for determining b_{Mit} , and it can certainly not be anything close to the expected outcome from a long-term perspective. If the long-term expected return for a benchmark portfolio is sufficiently negative, it will never be appropriate for an investor to want to hold it as part of their overall asset allocation.

Therefore, index funds should always have an advantage over hedge funds with respect to their expected cost for generating the returns from the benchmark portfolio and it will almost always be the case that index funds will have an advantage with respect to the actual costs. While this advantage will not quite constitute dominance in the spirit of the classic Modigliani and Miller examples for capital structure and dividend policy,³⁰ the scenarios required to avoid dominance due to the fee differential are sufficiently rare that it is reasonable to assume that index funds will dominate hedge funds as providers of passive returns for all time horizons meaningful from a long-term asset allocation perspective.

Because of the fee disadvantage, hedge funds should *never* be thought of as a source of passive returns, except for the return that can be earned from cash on the investment capital allocated to the manager. For a hedge fund manager, the appropriate prespecified asset allocation target value of b_{Mit} , with respect to all asset class benchmark portfolios, should be zero. In other words, the appropriate passive portfolio for hedge funds will simply be a 100 percent cash investment. The appropriate performance fee structure, at a minimum, should only pay a performance fee on returns in excess of the return available from cash. It should only be paid on returns in excess of those returns that are available without risk.

³⁰ See Modigliani and Miller (1958, 1959) and Miller and Modigliani (1961).

Flexibility of Hedge Funds Ideal for Active Management

Given the minimal assumptions required, the descriptive framework described in this paper is applicable for all investment managers. In particular, it can be used to demonstrate that any skill-based investing undertaken by traditional long-only active managers can be replicated in the more flexible structure of a hedge fund. Traditional long-only active managers will tend to differ from their hedge fund counterparts in the designation of the appropriate passive portfolio, in their fees, and in the potential constraints on their investment activity.

The difference in the appropriate passive portfolio will be in the designation of b_{Mit} . The assigned value of b_{Mit} for a traditional long-only active manager will almost always equal 100% or one. The active manager is evaluated against a passive benchmark, assuming the manager is fully invested, from an asset allocation perspective. Any deviation from “fully invested” is assumed to be an active management decision. Therefore, the appropriate Passive Portfolio Holdings for most traditional active managers is a 100% investment in the Benchmark Portfolio and no passive cash. Following from (13), this results in an active portfolio of

$$(43) \text{ Active Portfolio Holdings } (i,t) = \sum_k (w_{kit} - w_{Mkt}) (\text{Security } k) + w_{it} (\text{Cash})$$

Any deviation from a security’s weight in the benchmark portfolio is the result of an active decision. These deviations can be the result of a market timing decision, as reflected in θ_{Mi} , or due to specific underweighting or overweighting resulting from security selection.

As discussed above, the appropriate value of b_{Mit} for a hedge fund manager will almost always equal zero. As most hedge funds claim to be absolute return managers, the appropriate passive portfolio will earn the return available from riskless securities, which will always be positive. As this return can be achieved without risk, it is the appropriate comparison for a manager whose return, in excess of the riskless rate, is supposed to come purely from skill-based active investment decisions. It is also the appropriate comparison given the fee structure of most hedge funds, which receive performance incentives based on either absolute returns or returns in excess of the available return on cash or some other fixed hurdle rate. Therefore, the appropriate Passive Portfolio Holdings for most hedge fund managers is a 100% investment in passive cash. Again from (13), this results in an active portfolio of

$$(44) \text{ Active Portfolio Holdings } (i,t) = \sum_k w_{kit} (\text{Security } k) + (w_{it} - 1) (\text{Cash})$$

Any security holdings are the result of an active decision. These positions can be the result of a market timing decision, as reflected in θ_{Mit} , or due to specific underweighting or overweighting resulting from security selection.

Up to this point, we have imposed no restrictions on the composition of (43) or (44). In fact, in almost all cases, traditional, long-only active managers will be subject to much tighter constraints on their active portfolio holdings than would typically be the case for a hedge fund manager. For example, long-only managers are usually much more limited with respect to the universe of possible securities that are eligible for investment. Therefore, hedge funds will almost always be able to replicate the active investments of a traditional long-only manager, but the long-only manager will often not be able to replicate the investments of a hedge fund. And this discrepancy is even before taking into account leverage and short sales. Traditional active managers are usually prevented

from borrowing to finance positions or from being net short any security. Hedge funds, on the other hand, typically have the ability to use leverage or to go short.

The impact of the no borrowing constraint can be evaluated by combining (18) and (22) as follows

$$(45) \quad w_{it}(\Delta \text{ Cash}) = -\theta_{Mit} - w_{it}(\text{Long Portfolio}) - w_{it}(\text{Short Portfolio}) \geq (b_{Mit} - 1)$$

where the no borrowing constraint also requires that $b_{Mit} \leq 1$.

No borrowing requires that the incremental cash generated from or required for implementing the desired active management portfolio, $w_{it}(\Delta \text{ Cash})$, must be greater than or equal to $(b_{Mit} - 1)$ and must be nonnegative if $b_{Mit} = 1$. Any increased exposure to the benchmark must be paid for out of passive cash or underweight positions that exceed overweight positions in the aggregate. This will significantly limit a long-only manager's ability to undertake market timing. In the case of no market timing, $\theta_{Mit} = 0$, overweight positions in the aggregate must be paid for out of passive cash or from the proceeds generated from underweight positions. As most hedge fund managers are allowed to undertake some borrowing, they will be less constrained in their active investment decisions.

The constraint that a manager can not be net short any security requires that $w_{kit} \geq 0$ for all securities. Following from (13), this requires that the absolute value of the total underweighting from market timing and security selection can not exceed $(b_{Mit} * w_{Mkt})$. When $b_{Mit} = 1$, the total underweighting can not exceed the security's weighting in the benchmark portfolio. Again, most hedge funds are allowed to be net short and therefore, will be less constrained in their active investment decisions.

In most cases, hedge fund active managers are allowed to borrow in a constrained manner and to be net short most securities. Constraints may come in the form of maximum amount of borrowing that is allowed or in the aggregate size of the position. The impact of a cap on borrowing can be evaluated using (21).

$$(46) \quad W_{it}(\Delta \text{ Cash}) = -\theta_{Mit} - W_{it}(\text{Long Portfolio}) - W_{it}(\text{Short Portfolio}) \geq (b_{Mit} - 1) + \text{Borrowing Limit}$$

where the borrowing limit will reflect a negative amount of cash and will typically be proportional to the investment capital. It is unlikely that this constraint will ever be more binding than the comparable constraint for long-only managers and therefore, will almost always have less impact on the active investment decisions of a hedge fund manager than those of a long-only manager.

While hedge fund managers may have constraints on the absolute size of any individual short positions, by having the ability to go short, hedge funds will almost always have greater position flexibility than long-only managers. A constraint on the maximum position size requires that $w_{kit} \leq \text{Max}$ for all securities. Most traditional active managers are also subject to this constraint, although the constraint may also be on the maximum position size, relative to the weight in the benchmark portfolio. Again, constraints on maximum position size are usually at smaller levels for traditional managers than they are for hedge funds.

Given that hedge funds are almost always less constrained than traditional active managers, it will almost always be the case that a hedge fund can hold the exact same active portfolio as a traditional active manager. The only difference may be in the composition of the passive portfolio. But this difference will be known in advance and

can be offset in the investor's overall asset allocation. This can be accomplished either through the use of passive managers or through overlay strategies, as previously discussed.

Because a hedge fund can do anything a traditional active manager can, and more, it seems clear that hedge funds should be included in an investor's overall portfolio in the same way that any active manager is included. In fact, the increased flexibility available to hedge fund managers should allow those that possess active management skill to dominate managers with the same skill set, but less investment flexibility, at least before taking into account fee differences. The less constrained managers should be able to exploit the best return opportunities for active management while minimizing their exposure to unattractive sources of risk. Given the flexibility available to hedge fund managers, their portfolios can easily be structured to provide a pure play on skill-based active investments and therefore, all investments should be viewed as active decisions.

And the difference in fees between hedge funds and traditional long-only managers that are attributable to active management may be much smaller than is typically thought. While most traditional long-only managers do not charge performance fees, their management fees are much greater than those charged by their index fund counterparts. All of this incremental fee should be attributed to active management portion of the portfolio. But how much active management? If an active manager's portfolio only differs from the benchmark portfolio in ten percent of its holdings, then the incremental active management fee should effectively be allocated to ten percent of the total investment capital. Everything else equal, if a hedge fund is investing in active positions that in the aggregate are ten times larger than the traditional long-only manager, then ten times the fees are justified. Ultimately, the returns generated from active management

must justify the fees charged for active management and it will certainly be the case that the magnitude of the returns will depend on the size of the positions.

Of course, the increased flexibility also requires greater monitoring by the investor, both in terms of identifying active management skill and to reduce the possibility that the increased flexibility will be used by the manager to take risks that are different from or well in excess of what is expected by the investor. In the manager selection process, it will be important to determine how the manager can be expected to generate superior returns from skill-based active management. This includes identifying the return opportunities that the manager expects to exploit and identifying the manager's competitive advantage in exploiting the opportunity. Given the wide range of potential investment opportunities resulting from the flexibility available to hedge funds, it is especially important that the manager's investments match the manager's skill-based expertise and the risk management capabilities of the fund.

Given all of the empirical evidence that highlights the difficulty in achieving significantly positive and sustainable returns from alpha, the manager selection process is especially important. The investment flexibility available to hedge funds may finally provide an investment structure that allows top managers to consistently generate superior returns and the conventional fee structure for hedge funds should attract the best skill-based managers. However, given the high level of fees, and the ability of investors to invest in low-cost passive alternatives, all active managers, including hedge funds, should always be subject to the burden of proof with respect to their value-added in the active management allocation decision.

Summary

From the portfolio holdings of any investment manager, it will always be possible to separate active management decisions from the underlying passive portfolio. Given this separation, active management investments must be self-financing as they are not allocated any investment capital. This both requires and results in the portability of active management: skill-based active management can be combined with any of the asset classes that comprise the policy portfolio without restriction on the investment universe used to generate the alpha.

Given their fee structure and expenses, hedge funds will always be an inefficient source of passive returns. Therefore, hedge funds should only be considered as a pure play on skill-based active management. Given their flexibility, they may be the ideal structure for active management. While their fee structure may be expensive, such fees are likely to attract managers with the best market timing and security selection abilities. Given the difficulty in generating consistent and superior returns from active management, such fees are the cost of investing with the managers most likely to achieve such success. The challenge is to identify managers who can be expected to generate sustainable and superior returns, net of all management fees.

At a minimum, the appropriate performance evaluation and risk monitoring of hedge fund investments requires the disaggregation of the returns generated by the hedge fund manager to better understand the sources of return and sources of risk in the active portfolio. By disaggregating the portfolio holdings of the active manager and targeting the applicable themes, strategies, or risks inherent in the portfolio, investors can obtain important insights into the sources of return and the accompanying sources of risk that are generated from the manager's investment decisions. While the disaggregation suggested

in the descriptive framework proposed here will require much more information than is typically available to investors, in most cases, managers or prime brokers should be able to generate a daily time series of such disaggregated returns, using no more information than the minimum required by the manager or the prime broker for their own basic risk management. Even if only available with some time delay, such disaggregated returns should be able to provide substantial information for purposes of performance evaluation and risk monitoring, without requiring disclosure of exact security positions. Using such information, the proposed descriptive framework may provide a reasonable, and more user-friendly, compromise to full transparency.

Given that the appropriate role of hedge funds should be as a pure play on skill-based active management, it follows that they should not be thought of as a potential asset class. From an asset allocation perspective, an asset class should represent a potential source of return that offers investors a sufficient long-term return premium from a (relatively) passive investment. The portfolio representing the asset class should be investable and its composition should change minimally, if at all, over time. Other than identifying the common source of risk that is responsible for the expected return premium, the construction of the portfolio should not require any of the market-timing or security-selection skills associated with active management.

It is unlikely that any aggregation of hedge funds that will provide an attractive and feasible, passive investment vehicle. With such a portfolio, it will not be possible to isolate a stationary source of systematic risk that can be expected to earn an expected return premium. In addition, by definition, the return to active management in the aggregate must be zero, or negative when the costs and fees of the active management process are included. While it is possible that hedge fund managers can generate superior returns in the aggregate, at the expense of other active managers, it is certainly the case

that there are plenty of (large) hedge funds that are not capable of generating consistent and superior returns from active decisions. The inclusion of such funds in a portfolio of hedge funds designed to represent a passive investment vehicle will make it less likely that the portfolio of hedge funds will be able to offer a positive expected return premium. It will never make sense to invest with all possible hedge funds.

Instead, an optimally constructed portfolio of hedge funds should be considered as part of any active management overlay on the policy portfolio that results from the asset allocation decision. Such a portfolio should only include the best active managers, adjusting for differences in fees, and its composition should be expected to change over time. The manager selection and portfolio construction process itself will require skill-based analysis which can not only differentiate active from passive performance, but also assess the precise nature of the risks in the hedge fund manager's portfolio and the potential for those risks to generate a return premium. This skill-based analysis requires the application of skills similar to those used by the hedge fund managers themselves to make investment decisions.

Therefore, in the context of the asset allocation decision, a potential investment in hedge funds should not be treated as representing a new asset class or as a substitute for an alternative asset class investment. In this framework, hedge fund managers should be considered in the same way long-only active managers are considered. The decision to invest in hedge funds should be viewed as an active management decision where the potential investment is evaluated as an active overlay to the policy portfolio within the context of the asset allocation decision. The decision as to the amount to invest in hedge funds should be framed in terms of the desired amount of active management. This, in turn, will depend upon the expected return from skill-based investments and the characteristics of the risks that the investor is required to bear to achieve those expected

returns. These risks should be evaluated relative to the risks of the policy portfolio, which links the decision directly to the asset allocation decision.

Finally, the success of any hedge fund investment decision will not depend on the returns available from an investment spanning the universe of hedge funds, but will depend on the returns available from an optimally constructed portfolio of active managers: i.e., it will depend on the quality of manager selection process, the portfolio construction process, and the ongoing performance evaluation and risk monitoring of the active managers.

Table I: Capital Allocation and Leverage are Arbitrary

	Case #1		Case #2		Case #3	
	\$	%	\$	%	\$	%
Investment Capital	\$1,000		\$1,000		\$1,000	
<i>Expected Return</i>	\$92.0	9.20%	\$92.0	9.20%	\$92.0	9.20%
<i>Standard Deviation</i>	\$92.2	9.22%	\$92.2	9.22%	\$92.2	9.22%
Index Manager (IM) Capital	\$900		\$1,000		\$800	
<i>Expected Return</i>	\$72.0	8.00%	\$77.0	7.70%	\$67.0	8.38%
<i>Standard Deviation</i>	\$90.0	10.00%	\$90.0	9.00%	\$90.0	11.13%
IM Passive Investment Capital	\$900		\$1,000		\$800	
<i>Expected Return</i>	\$72.0	8.00%	\$77.0	7.70%	\$67.0	8.38%
<i>Standard Deviation</i>	\$90.0	10.00%	\$90.0	9.00%	\$90.0	11.13%
IM Benchmark Portfolio Allocation	\$900		900		900	
<i>Expected Return</i>	\$72.0	8.00%	\$72.0	8.00%	\$72.0	8.00%
<i>Standard Deviation</i>	\$90.0	10.00%	\$90.0	10.00%	\$90.0	10.00%
IM Cash Allocation	\$0		\$100		-\$100	
<i>Expected Return</i>	NA		\$5.0	5.00%	-\$5.0	-5.00%
<i>Standard Deviation</i>	NA		\$0.0	0.00%	\$0.0	0.00%
IM Active Investment Capital	\$0		\$0		\$0	
<i>Expected Return</i>	NA		NA		NA	
<i>Standard Deviation</i>	NA		NA		NA	
Active Manager (AM) Capital	\$100		\$0		\$200	
<i>Expected Return</i>	\$20.0	20.00%	\$15.0	NA	\$25.0	12.50%
<i>Standard Deviation</i>	\$20.0	20.00%	\$20.0	NA	\$20.0	10.00%
AM Passive Investment Capital	\$100		\$0		\$200	
<i>Expected Return</i>	\$5.0	5.00%	NA		\$10.0	5.00%
<i>Standard Deviation</i>	\$0.0	0.00%	NA		\$0.0	0.00%
AM Benchmark Portfolio Allocation	\$0		0		0	
<i>Expected Return</i>	NA		NA		NA	
<i>Standard Deviation</i>	NA		NA		NA	
AM Cash Allocation	\$100		\$0		\$200	
<i>Expected Return</i>	\$5.0	5.00%	NA		\$10.0	5.00%
<i>Standard Deviation</i>	\$0.0	0.00%	NA		\$0.0	0.00%
AM Active Investment Capital	\$0		\$0		\$0	
<i>Expected Return</i>	\$20.0	NA	\$20.0	NA	\$20.0	NA
<i>Standard Deviation</i>	\$20.0	NA	\$20.0	NA	\$20.0	NA
AM Active Portfolio Financing	-\$100		-\$100		-\$100	
<i>Expected Return</i>	-\$5.0	-5.00%	-\$5.0	-5.00%	-\$5.0	-5.00%
<i>Standard Deviation</i>	\$0.0	0.00%	\$0.0	0.00%	\$0.0	0.00%

NB: Correlation between Benchmark Portfolio Returns and Active Investment Returns = 0.

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